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Weighted inequalities for the Riesz potential on the sphere

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The fractional integral operator (or Riesz potential) of the classical Laplacian is given by

$$T_\sigma f(x) = \int_{\mathbb{R}^d} \frac{f(y)}{|x-y|^{d-\sigma}}\,dy, \quad 0 < \sigma < d.$$

Stein and Weiss (cf. [2]) proved that, under suitable conditions on the parameters, the fractional integral operator satisfies the inequality

$$\| |x|^{-a} T_\sigma f\|_{L^p(\mathbb{R}^d)} \le \| |x|^b f\|_{L^q(\mathbb{R}^d)}.$$

This result cannot be improved for general functions in $L^q(\mathbb{R}^d)$. However, De Napoli, Drelichman, and Durán (cf. [1]) extended the range of admissible power weights with f a radial function.

In this talk, we prove a version of the Stein-Weiss inequality in the setting of the d-dimensional sphere $\mathbb{S}^d$. In this space the role of the fractional integral T_σ is played in the sphere by the operator

$$A_\sigma f(\eta) = \frac{\Gamma(\sqrt{-L_\lambda} + \frac{1-\sigma}{2})}{\Gamma(\sqrt{-L_\lambda} + \frac{1+\sigma}{2})} f(\eta), \quad \eta \in \mathbb{S}^{d-1},$$

where $-L_\lambda$ is the conformal Laplacian on the sphere $L_\lambda = -\Delta_0 + \lambda^2$, $\lambda = (d-2)/2$. The operator A_σ can be written in terms of the Riesz potential on the sphere, so it is called the Riesz potential of the conformal Laplacian. So we shall prove the inequality

$$\|(|\eta - e_d||\eta + e_d|)^{-a} f A_\sigma f\|_{L^p(\mathbb{S}^{d-1})} \le C \|(|\eta - e_d||\eta + e_d|)^b\|_{L^q(\mathbb{S}^{d-1})}.$$

We also improve the range of admissible power weights for functions which are invariant under the action of the group $SO(d-1)$ on $\mathbb{S}^{d-1}$, the analogous to radial functions on the sphere.

References

[1] P. L. De Napoli, I. Drelichman, and R. G. Durán, On weighted inequalities for fractional integrals of radial functions, *Illinois J. Math.* **55** (2011), 575–587.

[2] E. M. Stein and G. Weiss, Fractional integrals on n-dimensional Euclidean space, *J. Math. Mech.* **7** (1958), 503–514.

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