

Christoffel transformations for matrix orthogonal polynomials in the real line

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Given a matrix polynomial $W(x)$, matrix bi-orthogonal polynomials with respect to the sesquilinear form

$$\langle P(x), Q(x) \rangle_W = \int P(x)W(x)d\mu(x)(Q(x))^\top, \quad P, Q \in \mathbb{R}^{p \times p}[x],$$

where $\mu(x)$ is a matrix of Borel measures supported in some infinite subset of the real line, are considered. Connection formulas between the sequences of matrix bi-orthogonal polynomials with respect to $\langle \cdot, \cdot \rangle_W$ and matrix polynomials orthogonal with respect to $\mu(x)$ are presented. In particular, for the case of nonsingular leading coefficients of the perturbation matrix polynomial $W(x)$ we present a generalization of the Christoffel formula constructed in terms of the Jordan chains of $W(x)$. For perturbations with a singular leading coefficient several examples by Durán et al are revisited.

References

- [1] C. ÁLVAREZ-FERNÁNDEZ, G. ARIZNABARRETA, J. C. GARCÍA-ARDILA, M. MAÑAS, F. MARCELLÁN, *Christoffel transformations for matrix orthogonal polynomials in the real line and the non-Abelian 2D Toda lattice hierarchy*, Internat. Math. Res. Notices, in press. arXiv:1511.04771v2 [math.CA].

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