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On the eigenvalues of Aharonov-Bohm operators with varying poles

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Let Ω be an open, simply connected set in $\mathbb{R}^2$. For $a = (a_1, a_2) \in \Omega$, we consider the following magnetic operator: $(i\nabla + A_a)^2$ acting on functions u with zero boundary conditions on $\partial\Omega$. The magnetic potential A_a, singular at the point a and having circulation α around a, has the following form

$$Aa(x_1, x_2) = \alpha \left(-\frac{x_2 - a_2}{|x - a|^2}, \frac{x_1 - a_1}{|x - a|^2} \right).$$

It corresponds to a magnetic field, being a multiple of the delta-Dirac, orthogonal to Ω at a. Those two objects are related to the Bohm-Aharonov effect. More particularly, we are interested in the spectrum of this operator when the pole a moves in Ω and eventually approaches the boundary of Ω. For all circulation $\alpha \in (0, 1)$, we have proved the continuity of the magnetic eigenvalue with respect to a; and differentiability as long as the eigenvalue remains simple. We also prove the continuity up to the boundary of Ω: as a converges to the boundary of Ω, the k-th magnetic eigenvalue converges to the k-th eigenvalue of the Laplacian with Dirichlet boundary conditions. This implies in particular that the magnetic eigenvalue (as a function of a) always has an extremal point in Ω. In the case of half-integer circulation $\alpha = 1/2$, this problem is re- lated to a Laplacian problem in a double covering. For this reason, we can deduce the behaviour of the nodal lines of the magnetic eigenfunctions: far away from the singularity a, the eigenfunction behaves like an eigenfunction of the Laplacian, while at the singular point a, it has an odd number of nodal lines. Then, we study additional properties of extremal interior points. We will show that the rate of convergence of the magnetic eigenvalues when a goes to some fix point b depends on the number of nodal lines of the corresponding magnetic eigenfunction with pole at b. For this, we use inversion theorem. We can also prove a rate of convergence of the eigenvalues when the pole a approaches the boundary. This requires completely different techniques. We use then an Algrem?s type formula. Those results are corroborated by numerical simulations in the case when Ω is an angular sector or a square.
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