Additivity of 2-local and weak-2-local maps on \(C^*\)-algebras

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Additivity of certain maps defined on \(C^*\)-algebras seems to be closely related to other algebraic properties determined by the Algebraic-Analytic structure of \(C^*\)-algebras. That is the case of 2-local and weak-2-local maps defined by 2-local actions of derivations and \(*\)-homomorphisms on \(C^*\)-algebras.

We shall present the latest advances on 2-local and weak-2-local maps on \(C^*\)-algebras. Let \(S\) be a subset of the space \(L(X,Y)\) of all linear maps between Banach spaces \(X\) and \(Y\). We recall that a (non-necessarily linear nor continuous) mapping \(\Delta : X \rightarrow Y\) is a 2-local \(S\) map (respectively, a weak-2-local \(S\) map) if for each \(x, y \in X\) (respectively, if for each \(x, y \in X\) and \(\phi \in Y^*\)), there exists \(T_{x,y} \in S\), depending on \(x\) and \(y\) (respectively, \(T_{x,y,\phi} \in S\), depending on \(x, y\) and \(\phi\)), satisfying

\[
\Delta(x) = T_{x,y}(x), \quad \text{and} \quad \phi \Delta(y) = T_{x,y}(y)
\]

(respectively,
\[
\phi \Delta(x) = \phi T_{x,y,\phi}(x), \quad \text{and} \quad \phi \Delta(y) = \phi T_{x,y,\phi}(y).
\]

We shall mainly focus on the case in which \(X\) and \(Y\) are \(C^*\)-algebras and \(S\) is the set of all derivations on \(A\) or the set of all \(*\)-homomorphisms from \(A\) to \(B\), or simply the set of all symmetric linear maps between \(A\) and \(B\). We shall derive generalizations of well known, and significant, results due to Kowalski and Słodkowski [2], Šemrl [3], and Ayupov and Kudaybergenov [1].

References


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Author partially supported by the Spanish Ministry of Economy and Competitiveness and European Regional Development Fund project no. MTM2014-58984-P and Junta de Andalucía grant FQM375.