On linear refinements of geometric inequalities

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The Brunn-Minkowski inequality is one of the most powerful theorems in Convex Geometric Analysis and beyond: it implies, among others, very relevant results such as the isoperimetric and Urysohn inequalities (see e.g. [3, s. 7.2]). It can be summarized by stating that the volume (the Lebesgue measure in \( \mathbb{R}^n \)) is \((1/n)\)-concave, i.e.,

\[
\text{vol}
((1 - \lambda)K + \lambda L)^{1/n} \geq (1 - \lambda)\text{vol}(K)^{1/n} + \lambda\text{vol}(L)^{1/n},
\]

for all convex bodies \( K, L \) and \( \lambda \in (0, 1) \).

Moreover, it is well-known that this exponent is necessary and further the best possible that one may expect. However, a classical result by Bonnesen asserts that if the convex bodies have a common volume projection onto a hyperplane, then the volume itself is a concave function, which enhances the statement of Brunn-Minkowski’s theorem.

Here we will show that some other classical inequalities such as the Prékopa-Leindler inequality, the Minkowski first inequality or the isoperimetric inequality share this linear demeanor (under assumptions on projections/sections) with the Brunn-Minkowski inequality. Moreover, we will show that the above-mentioned behavior remains true in the setting of the Gauss Space, i.e., the \( n \)-dimensional Euclidean space \( \mathbb{R}^n \) endowed with the standard gaussian measure, a fact that will allow us to obtain further Brunn-Minkowski type inequalities for the Gauss measure.

The content of this contribution is based on the works [1, 2, 4].

References


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