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On linear refinements of geometric inequalities

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The Brunn-Minkowski inequality is one of the most powerful theorems in Convex Geometric Analysis and beyond: it implies, among others, very relevant results such as the isoperimetric and Urysohn inequalities (see e.g. [3, s. 7.2]). It can be summarized by stating that the volume (the Lebesgue measure in $\mathbb{R}^n$) is $(1/n)$-concave, i.e.,

$$\mathrm{vol}\big((1-\lambda)K+\lambda L\big)^{1/n} \geq (1-\lambda)\mathrm{vol}(K)^{1/n}+\lambda\mathrm{vol}(L)^{1/n},$$

for all convex bodies K, L and $\lambda \in (0,1)$.

Moreover, it is well-known that this exponent is necessary and further the best possible that one may expect. However, a classical result by Bonnesen asserts that if the convex bodies have a common volume projection onto a hyperplane, then the volume itself is a concave function, which enhances the statement of Brunn-Minkowski's theorem.

Here we will show that some other classical inequalities such as the *Prékopa-Leindler inequality*, the *Minkowski first inequality* or *the isoperimetric inequality* share this linear demeanor (under assumptions on projections/sections) with the Brunn-Minkowski inequality. Moreover, we will show that the above-mentioned behavior remains true in the setting of the Gauss Space, i.e., the n-dimensional Euclidean space $\mathbb{R}^n$ endowed with the standard gaussian measure, a fact that will allow us to obtain further Brunn-Minkowski type inequalities for the Gauss measure.

The content of this contribution is based on the works [1, 2, 4].

References

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