Superstable expansions of \((\mathbb{Z}, +, 0)\)

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In [1] and in [2], it is proved that the structure \((\mathbb{Z}, +, 0, \Pi_q)\) is superstable, where \(\Pi_q\) is interpreted as the set of powers of a fixed natural number \(q \geq 2\). In this work, we generalize this result to a wider class of sequences of natural numbers. Specifically, we prove that under one of the following two conditions on a sequence \(R = (r_n)_{n \in \mathbb{N}}\) of natural numbers, the structure \((\mathbb{Z}, +, 0, R)\) is superstable:

1. \(\lim_{n \to \infty} \frac{r_{n+1}}{r_n} = \infty\) and \(R\) is ultimately periodic modulo \(m\), for any natural number \(m > 1\);
2. \(R\) follows a linear recurrence relation and there exists a positive real number \(\theta\) such that \(\lim_{n \to \infty} \frac{r_n}{\theta^n} \in \mathbb{R}\setminus\{0\}\).

Our strategy is the following. We first give a universal axiomatization of the (first order) theory of \((\mathbb{Z}, +, 0, R)\) in a reasonable language so as to express the behaviour of equations satisfied by elements of \(R\). We then prove that this theory is model-complete and, since our axiomatization is universal, deduce quantifier elimination. A counting of types argument then shows superstability.

References


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