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Extensions of Minkowski's theorem on successive minima

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Let K be a 0-symmetric convex body, i.e., a compact and convex set satisfying that $K = -K$, in the n-dimensional Euclidean space $\mathbb{R}^n$, and let $\mathbb{Z}^n$ denote the integer lattice. The well-known *Minkowski 2nd theorem* in the Geometry of Numbers (see e.g. [2]) provides optimal upper and lower bounds for the volume of K in terms of its *successive minima*:

$$\frac{1}{n!}\prod_{i=1}^{n}\frac{2}{\lambda_i(K,\mathbb{Z}^n)} \leq \mathrm{vol}(K) \leq \prod_{i=1}^{n}\frac{2}{\lambda_i(K,\mathbb{Z}^n)},$$

and both bounds are best possible. Here, $\lambda_i(K,\mathbb{Z}^n) = \min\{\lambda > 0 : \dim(\lambda K \cap \mathbb{Z}^n) \geq i\}$ is the i-th successive minimum of K with respect to the integer lattice, $1 \leq i \leq n$, and $\mathrm{vol}(K)$ denotes the volume (Lebesgue measure) of K.

Since this important theorem was discovered, many mathematicians have worked on new proofs and extensions of it. We aim to make a brief historical tour on this inequality and its generalizations. Then we will show new analogs of the theorem from two different points of view (see [1]): either relaxing the symmetry condition, or replacing the volume functional by the surface area. Thus, if we assume for instance that the centroid lies at the origin, then the following tight inequality can be proved:

$$\frac{n+1}{n!}\prod_{i=1}^{n}\frac{1}{\lambda_i(K,\mathbb{Z}^n)} \leq \mathrm{vol}(K).$$

References

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[2] H. Minkowski, *Geometrie der Zahlen.* Teubner, Leipzig-Berlin, 1896, reprinted by Johnson Reprint Corp., New York, 1968.

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