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Domination via (min,+) algebra

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Perfect codes have played a central role in the development of error-correcting codes theory. A code in a graph is vertex set such that any two vertices in it are at distance at least 3. If, in addition, every vertex not in the code has a neighbor in it, the code is called perfect. Therefore a perfect code is an independent dominating set a such every vertex not in the set has a unique neighbor in it. The existence of this type of codes is not guaranteed in every graph and it has been extensively studied. For instance it is well known that the cartesian product $P_m \square P_n$ of two paths has no perfect code unless $m = n = 4$ or $m = 2, n = 2k + 1$. In these cases a less demanding construction could be keeping domination and independence but admitting at most two neighbors, for vertices not in the dominating set. A set satisfying these conditions is called independent $[1, 2]$-set and we have shown that every grid has one of them. Then the question of calculating $i_{[1,2]}(P_m \square P_n)$ the cardinal of such a set with minimum size arises.

The calculation of domination parameters in grids has proved to be a difficult task. Indeed finding $\gamma(P_m \square P_n)$, the cardinal of a minimum dominating set in the grid, was an open problem for almost 30 years, since it was first studied in 1984 in relation with the still open Vizing's Conjecture. An important milestone on the way to the solution is the upper bound $\gamma(P_m \square P_n) \leq \lfloor \frac{(m+2)(n+2)}{5} \rfloor - 4$, for $m, n \geq 8$, found in 1992, like the conjecture about that equality is achieved in case $m, n \geq 16$. This conjecture was finally confirmed in 2011, ending the study of this problem. Meantime different efforts were made to calculate exact values of $\gamma(P_m \square P_n)$, for fixed small m and every $n \geq m$.

Among the different techniques used to address this problem, we would like to focus on a dynamic programming algorithm developed by applying the (min,+) matrix multiplication. Values of $\gamma(P_m \square P_n)$ for $m \leq 19$ and $n \geq m$ were obtained with this algorithm and we have adapted it to compute $i_{[1,2]}(P_m \square P_n)$ for fixed m and $n \geq m$. The resulting algorithm can be theoretically applied in every grid, however the long running time needed make them useful just on grids of small size, in our case $m \leq 13$ and $n \geq m$. On the other hand $i_{[1,2]}(P_m \square P_n)$ can be obtained with a quasi-regular pattern in grids with $14 \leq m \leq n$, resembling the above-mentioned construction for the upper bound of the domination number. In addition this pattern explicitly provides an independent $[1, 2]$-set of minimum size.

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