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The Manickam-Miklos-Singhi Conjecture in Partial Linear Spaces

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Let $\mathcal{P}$ be a set of *points*, $\mathcal{L}$ a set of *lines* and $\mathsf{I} \subseteq \mathcal{P} \times \mathcal{L}$ an *incidence relation*. Define real-valued weight function $f : \mathcal{P} \to \mathbb{R}$ such that $\sum_{p \in \mathcal{P}} f(p) = 0$. The weight of a line ℓ is $\sum_{p \in \mathcal{P}_\ell} f(p)$, where $\mathcal{P}_\ell$ is the set of points p with $p \,\mathsf{I}\, \ell$.

Consider the case that $\mathcal{P}$ is the set $N = \{1, 2, \ldots, n\}$ and $\mathcal{L}$ is the set of all k-subsets of N. In 1988 Manickam and Singhi conjectured that if $n \geq 4k$, then the number of lines with nonnegative weight is at least $\binom{n-1}{k-1}$. They conjectured the same for the case, where $\mathcal{P}$ is the set of 1-dimensional subspaces of an n-dimensional vector space V over a finite field with q, and $\mathcal{L}$ is the set of k-dimensional subspaces of V. These conjectures are called MMS conjectures.

We will discuss various results for variants of the MMS conjecture for sets, vectors spaces and partial linear spaces. Furthermore, we will point out connections between the MMS conjecture, so-called Erdős-Ko-Rado theorems and spreads.

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