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The finiteness threshold width of lattice polytopes

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A lattice d-*polytope* $P \subset \mathbb{R}^d$ is the convex hull of finitely many points from $\mathbb{Z}^d$. The *size* of P is the number of integer points it contains, and the *width* of P is the minimum, over all non-constant integer linear functionals f, of the length of the interval $f(P)$. For each $d \geq 3$ and each $n \geq d+1$, there exist infinitely many lattice d-polytopes of size n.

In [1] we prove that there exists a constant $w^\infty(d)$, depending solely on d, such that all but finitely many d-polytopes of size n have width at most $w^\infty(d)$. We call $w^\infty(d)$ the *finiteness threshold width*. We show that $w^\infty(d)$ equals the maximum width of a lattice hollow $(d-1)$-polytope with infinitely many d-dimensional lifts of the same size and width. This allows us to prove that $d-2 \leq w^\infty(d) \leq O\left(d^{3/2}\right)$ and, more particularly, that $w^\infty(4) = 2$ and $w^\infty(5) \geq 4$. Blanco and Santos had already determined the value $w^\infty(3) = 1$ [2].

References

[1] M. Blanco, C. Haase, J. Hofmann and F. Santos, The finiteness threshold width of lattice polytopes, in preparation.

[2] M. Blanco and F. Santos, Lattice 3-polytopes with few lattice points. Preprint, 23 pages, September 2014, revised November 2015, `arXiv:1409.6701`. Accepted in *SIAM J. Discrete Math.*

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