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Dynamics and interpretation of some integrable systems via matrix orthogonal polynomials

Ana Mendes[1], Amílcar Branquinho[2], Ana Foulquié Moreno[3]

In this work we characterize a Toda type lattice in terms of a family of matrix polynomials orthogonal with respect to a complex matrix measure. In order to study the solution of this dynamical system we give explicit expressions for the Weyl function, generalized Markov function, and we also obtain, under some conditions, a representation of the vector of linear functionals associated with this system. We also prove a Lax type theorem for the point spectrum of the Jacobi operator associated with a Toda type lattice.

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[1]Department of Mathematics, School of Technology and Management, Polytechnic Institute of Leiria, 2411-901 Leiria, Portugal
`aimendes@ipleiria.pt`

[2]CMUC and Department of Mathematics, University of Coimbra, Apartado ,
3008, EC Santa Cruz, 3001-501 Coimbra, Portugal
ajplb@mat.uc.pt

[3]CIDMA and Department of Mathematics, University of Aveiro, 3810-193
Aveiro, Portugal
foulquie@ua.pt