II Joint Conference of the Belgian, Royal Spanish and Luxembourg Mathematical Societies
Logroño, June 6–8, 2016

Heat kernel formulae and the Brownian bridge to a submanifold

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Heat kernels are important for a variety of reasons. They are connected to the study of the heat equation, can be used as analytic tools and capture certain geometric and topological properties of the underlying space. For Riemannian manifolds with a pole, the minimal heat kernel has a probabilistic formula due to K.D. Elworthy and A. Truman (cf. [1]). By a joint with X.-M. Li (cf. [2]), this can be adapted to also provide formulae for the derivatives of the heat kernel.

In this talk, we present a more general formula, for the minimal heat kernel on any complete Riemannian manifold. This object coincides with the transition densities of Brownian motion, whose construction on a manifold we will review. The Brownian bridge is given by conditioning Brownian motion to hit a fixed point at a fixed positive time. We extend this concept by replacing the fixed point with a submanifold and use our formula to derive lower bounds, an asymptotic relation and derivative estimates for the conditional measure.

The motivation for this is the desire to extend the analysis of path and loop spaces to measures on paths which terminate on a submanifold, the intention being to study the relationship between the geometry of the path space, the intrinsic geometry of the ambient manifold and the extrinsic geometry of the submanifold.

References

[1] K. D. Elworthy and A. Truman, The diffusion equation and classical mechanics: an elementary formula, *Stochastic processes in quantum theory and statistical physics* **173** (1982), 136–146.

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