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A journey in the zoo of Turing patterns

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Self-organizing phenomena are widespread in Nature and have been studied for a long time in various domains, be it in physics, chemistry, biology, ecology, neurophysiology, to name a few [1]. Despite the rich literature on the subject, there is still need for understanding, analyzing and predicting their behaviors.

They are commonly based on local interaction rules which determine the creation and destruction of the entities at every place, upon which a diffusion process determines the migration of the components. For this reason reaction-diffusion systems are a common framework of modeling such systems [2].

In a 1952 article in biomathematics, Turing considered a two-species model of morphogenesis [3]. For the first time, he established the conditions for a stable spatially homogeneous state, to migrate towards a new heterogeneous, spatially patched, equilibrium under the driving effect of diffusion, at odd with the idea that diffusion is a source of homogeneity. Even though the explanation for morphogenesis has evolved and now relies more on genetic programming, many actual results are based or inspired form this pioneering work. The so-called *Turing instabilities*, or *Turing patterns*, help explain by a simple means the emergence of self-organized collective patterns.

The geometry of the underlying support where the reaction-diffusion acts, plays a relevant role in the patterned outcome, it can be because of the non flat geometry [4] (possibly growing) [5] or because of its anisotropy [6]. Pushing to the extreme the discreteness of the space, scholars have considered reaction-diffusion systems on complex networks; reactions occur at each node and then products are displaced across the network using the available links, thus possibly exhibiting Turing patterns [7].

The aim of this talk will be to introduce some of the recent developments that improve the classical ones by Turing to the framework of more general complex networks supports, for instance multiplex [8, 9] and cartesian product networks [10], or with more generic reaction parts, e.g. involving delays.

References

[1] G. Nicolis and I. Prigogine, *Self-Organization in Nonequilibrium Systems: From Dissipative Structures to Order through Fluctuations*, Wiley (1977).

[2] P. Grindrod, *Patterns and waves: The theory and applications of reaction-diffusion equations*, Clarendon Press Oxford, (1991).

[3] A. Turing, The Chemical Basis of Morphogenesis, *Phils Trans R Soc London Ser B*, **237**, (1952), 37.

[4] C. Varea, J.L. Aragón and R.A. Barrio, Turing patterns on a sphere, *Phys. Rev. E*, **60**, (1999), 4588.

[5] R.G. Plaza et al, The Effect of Growth and Curvature on Pattern Formation, *J. of Dynamics and Differential Equations*, **16**, (4), (2004), 1093.

[6] D.M. Busiello et al, Pattern formation for reactive species undergoing anisotropic diffusion, *Eur. Phys. J. B*, **88**, (2015), 88.

[7] H. Nakao and A.S. Mikhailov, Turing patterns in network-organized activator-inhibitor systems, *Nature Physics*, **6**, (2010), 544.

[8] M. Asllani et al., Turing patterns in network-organized activator-inhibitor systems, *Physical Review E*, **90**, (2014), 042814.

[9] N. E. Kouvaris, S. Hata, A. Díaz-Guilera, Pattern formation in multiplex networks, *Scientific Report*, **5**, (2015), 10840.

[10] M. Asllani et al., Turing instabilities on Cartesian product networks,*Scientific Report*, **5**, (2015), 12927.

[11] J. Petit, T. Carletti, M. Asllani and D. Fanelli, Delay-induced Turing-like waves for one-species reaction-diffusion model on a network, *European Physical Letter*, **111**, (2015), 58002

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