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Coexistence of hyperchaos and chaos: a Computer-assisted proof

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It has recently been reported that it is quite difficult to distinguish between chaos and hyperchaos in numerical simulations which are frequently "noisy". In this talk we show that, for the classical 4D Rössler model, the coexistence of two invariant sets with different nature (a global hyperchaotic invariant set and a chaotic attractor) and the homoclinic and heteroclinic connections between their unstable periodic orbits give rise to long hyperchaotic transient behavior, and therefore it provides a mechanism for noisy simulations [1].

Moreover, the existence of several hyperchaotic sets provides an explanation of the smooth change from chaotic to hyperchaotic attractors due to the appearance of new heteroclinic connections among them, and so the joining of the different sets gives rise to slightly bigger and slightly more hyperchaotic attractors in the sense that the second Lyapunov exponent grows a little. The same phenomena are expected in other 4D and higher dimensional systems.

The Computer-assisted proof of this coexistence of chaotic and hyperchaotic behaviors combines topological and smooth methods with rigorous numerical computations [2]. The existence of (hyper)chaotic sets is proved by the method of covering relations and cone conditions [3].

References

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