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# Duality on value semigroups

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The semigroup of values is a classical combinatorial invariant associated to a curve singularity. It is defined by taking all regular elements of the corresponding ring to the integral closure and taking a multivaluation. For a curve singularity with $r$ branches the semigroup of values is a submonoid of $\mathbb{N}^r$. In the irreducible case it is a numerical semigroup, otherwise it is not even finitely generated. Due to Lejeune-Jalabert and Zariski, this value semigroup determines the topological type of plane complex curves. As observed by Kunz in the irreducible case, the Gorenstein property of a curve singularity is equivalent to a symmetry of gaps and non-gaps in the (numerical) value semigroup. Delgado generalized this result to the reducible case introducing a non-obvious notion of symmetry of a semigroup. A canonical (fractional) ideal on a curve singularity defines a duality on fractional ideals. On the other hand taking multivaluations as above associates to any fractional ideal a value semigroup ideal. Such value semigroup ideals satisfy certain natural axioms defining the class of so-called good semigroup ideals. Barucci, D'Anna and Frberg gave an example of a good semigroup that does not come from a ring. Extending Delgado's symmetry result, D'Anna described the value semigroup ideals of canonical ideals. In the Gorenstein case, Delphine Pol described the value semigroup ideal of duals. Unifying the work of D'Anna and Pol we establish a purely combinatorial duality on good semigroup ideals that mirrors the duality on fractional ideals. The talk is based on joint work with Philipp Korell and Laura Tozzo.

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