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Liouville type equations with sign-change data

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In this talk, we study the existence of solutions of the following problem

$$-\Delta_g u = \lambda \left(\frac{Ke^u}{\int_\Sigma Ke^u dV_g} - 1 \right) \text{ in } \Sigma \tag{1}$$

where Σ is a compact surface without boundary, equipped with a Riemannian metric g, Δ_g is the Laplace-Beltrami operator, $\lambda > 0$, $Vol_g(\Sigma) = \int_\Sigma 1 dV_g = 1$ and K is a function defined in Σ. It is well-known that (1) has a geometric meaning, arising in the assigned Gauss curvature problem. From a variational point of view, the problem has been extensively studied, see for example [4], for the regular case, and [1] for the singular one, under the assumption that K is positive.

However, imposing additional hypotheses on ∇K in the region where $K = 0$, based of a moving-planes argument and a priori estimates, it is possible give a existence result for the sign-changing case for the sphere, [2], and for general surfaces [3].

References

[1] D. Bartolucci, F. De Marchis, A. Malchiodi, *Supercritical conformal metrics with conical singularities,* Int. Mat. Res. Not. 24 (2011), pp. 5625–5643.

[2] F. De Marchis, R. López-Soriano *Existence and Non Existence Results for the singular Nirenberg problem,* Calc. Var. Partial Differential Equations, 55 (2016), no. 2, 55:36.

[3] F. De Marchis, R. López-Soriano *work in progress.*

[4] Z. Djadli, *Existence result for the mean field problem on Riemann surfaces of all genuses,* Commun. Contemp. Math. 10 (2008), no. 2, 205–220.

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