

Units in Integral Group Rings via Fundamental Domains and Hyperbolic Geometry

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The motivation of this work is the investigation on the unit group of an integral group ring $\mathcal{U}(\mathbb{Z}G)$ for a finite group G . By the Wedderburn-Artin Theorem, the study of $\mathcal{U}(\mathbb{Z}G)$ may be reduced, up to commensurability, to the study of $\mathrm{SL}_n(\mathcal{O})$ for $n \geq 1$ and \mathcal{O} an order in some division ring D . There exists descriptions of a finite set of generators for a subgroup of finite index in $\mathrm{SL}_n(\mathcal{O})$ for a large number of cases. Excluded from this result are the so-called exceptional components of $\mathbb{Q}G$.

Our work consists in finding a presentation, for $\mathrm{SL}_n(\mathcal{O})$ associated to some of these exceptional components. In all the cases we treat, the group $\mathrm{SL}_n(\mathcal{O})$ has a discontinuous action on hyperbolic space of dimension 2 or 3, on hyperbolic space of higher dimensions, or on some product of hyperbolic spaces. By constructing fundamental domains for these discontinuous actions, we get generators for the groups in question.

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