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On the Prime Graph Question for symmetric groups

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Given a finite group G, the integral group ring $\mathbb{Z}G$ of G over the integers $\mathbb{Z}$, is defined as the ring of all linear combinations of the form $u = \sum_{g \in G} u_g g$, where the coefficients u_g are integers. The addition is defined componentwise and the multiplication is the extension of the group multiplication. Let $\mathcal{U}_1(\mathbb{Z}G)$ denote the group of normalized units of $\mathbb{Z}G$. There are many interesting questions around such a group of units, specially for units of finite order. An example of this is the Prime Graph Question. The prime graph $\Gamma(G)$ of a group G, which was introduced by Gruenberg and Kegel, is the graph having as vertices those rational primes p for which there exists an element of this order in the group G and two vertices p and q are connected by an edge if there is an element of order pq in G. Then the Prime Graph Question asks whether $\Gamma(\mathcal{U}_1(\mathbb{Z}G)) = \Gamma(G)$ for a finite group G.

Let G be an almost simple groups with socle A_n, the alternating group of degree n. We prove that there is a unit of order pq in the integral group ring of G if and only if there is an element of that order in G provided p and q are primes greater than $\frac{n}{3}$. We combine this with some explicit computations to verify the Prime Graph Question for all almost simple groups with socle A_n if $n \leq 17$.

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