

## On bialgebroids and warpings

Mitchell Buckley<sup>1</sup>, Joost Vercruyse<sup>2</sup>

There are many different characterisations of bialgebras and their generalisations. Each characterisation lends itself to different weaker notions of bialgebra and alternative conditions for identifying Hopf algebras among bialgebras. In particular, a bialgebroid  $A$  over base algebra  $R$  can be characterised as:

1. an opmonoidal monad on  ${}_R\text{Mod}_R$  possessing a right adjoint [1, 2];
2. a closed right skew monoidal structure on  $\text{Mod}_R$  with unit  $R$  [3]; or
3. a monoidal structure on  $\text{Mod}_A$  which is strictly preserved by the forgetful functor to  ${}_R\text{Mod}_R$  [4].

It is not trivial to show that these are equivalent, yet the underlying relationships follow from abstract arguments concerning monads, warpings, and skew monoidal categories.

Without reproducing the above equivalences entirely, we show how the bases of these results rely on simple categorical constructions. In particular we highlight the role of warpings and their relationship with fusion operators and opmonoidal monads. This work is still in progress.

### References

- [1] K. SZLACHÁNYI, The monoidal Eilenberg-Moore construction and bialgebroids, *Journal of Pure and Applied Algebra*, **182(2–3)** (2003), 287–315.
- [2] G. BÖHM, Hopf algebroids, in *Handbook of algebra. Vol. 6*, Elsevier, Amsterdam, 2009.
- [3] K. SZLACHÁNYI, Skew-monoidal categories and bialgebroids, *Advances in Mathematics*, **231(3–4)**, (2012), 1694 – 1730.
- [4] P. SCHAUENBURG, Bialgebras over noncommutative rings and a structure theorem for Hopf bimodules, *Appl. Categ. Structures*, **6(2)** (1998), 193–222.

<sup>1</sup>Département de Mathématiques, Faculté des sciences, Université Libre de Bruxelles, Boulevard du Triomphe, B-1050 Bruxelles, Belgium  
mitchell.alan.buckley@gmail.com

<sup>2</sup>Département de Mathématiques, Faculté des sciences, Université Libre de Bruxelles, Boulevard du Triomphe, B-1050 Bruxelles, Belgium  
jvercruy@ulb.ac.be