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Constant mean curvature surfaces in Riemannian product spaces.

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The geometry and topology of 3-manifolds with non-negative curvature has been an active field of research during the last century. One of their most insteresting features is the fact that they generally admit compact (without boundary) embedded minimal surfaces. Many authors have contributed to the study of compact minimal surfaces in order to understand the geometry and topology of the 3-manifold. We emphasize the work of Lawson, who showed the existence of embedded compact minimal orientable surfaces of any topological type in the 3-sphere as well as the first known examples of double periodic constant mean curvature surfaces in $\mathbb{R}^3$. The technique he developed is nowadays known as the *conjugate Plateau* and has been fruitfully used in the literature.

We will show how the conjugate Plateau construction can be developed in the Riemannian products $\mathbb{S}^2 \times \mathbb{R}$ and $\mathbb{H}^2 \times \mathbb{R}$ addressing its limitations and particularities. Then, we will talk about some advances in the understanding of the surfaces with constant mean curvature (including minimal ones) in those spaces [1, 2].

Finally, we will also describe a different method to produce minimal surfaces in the aforementioned spaces via the generalized Gauss map and their relation with minimal surfaces in the 3-sphere and anti-De Sitter space [3, 4].

References

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