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Lagrangian mean curvature flow of Hopf tori

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The mean curvature flow (MCF) of a smooth immersion $F_0 : M^n \to \mathbb{R}^m$ is a family of immersions $F : M \times [0,T) \to \mathbb{R}^m$ parameterized by t, and satisfying

$$\frac{dF}{dt}(p,t) = H(p,t), \qquad F_0 = F(\cdot, 0), \tag{$\star$}$$

where $H(p,t)$ denotes the mean curvature vector of $F_t(M)$ at $F_t(p) = F(p,t)$, and $[0,T)$ is the maximal time interval such that $(\star)$ holds. Unless the flow has an eternal solution (i.e., it is defined for all t), MCF fails to exist after a finite time, giving rise to a singularity.

These singularities are classified depending on the blow-up rate of the second fundamental form. The so-called Type I singularities are those such that the blow-up of the second fundamental form is best controlled; the remaining singularities are known as Type II singularities. Moreover, by the Huisken's monotonocity formula, these Type I singularities look like self-similar contracting solutions after an appropriate rescaling.

On the other hand, the Huisken's classical result: *if the initial hypersurface is uniformly convex, then its MCF converges to a round point in finite time*, and the fact that Lagrangian self-shrinking spheres do not exist, motivates the open question posed by Neves about *finding a condition on a Lagrangian torus in $\mathbb{C}^2$, which implies that the Lagrangian mean curvature flow $(M_t)_{0<t<T}$ will become extinct at time T and, after rescaling, M_t converges to the Clifford torus* (cf. [2]). In fact, in the Lagrangian context, the Clifford torus is the most regular example of a compact self-shrinker for Lagrangian MCF in complex Euclidean plane.

In this talk we will answer Neves question by describing the evolution by MCF of a Hopf torus M_0 dividing $\mathbb{S}^3(R_0)$ in two components of equal volume. We will also give examples of Lagrangian surfaces developing Type II singularities (cf. [1]).

References

[1] I. Castro, A. M. Lerma, and V. Miquel, Evolution by mean curvature flow of Lagrangian spherical surfaces in complex Euclidean plane, arXiv:1603.03229 [math.DG].

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