Additivity of 2-local and weak-2-local maps on C*-algebras

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Additivity of certain maps defined on C*-algebras seems to be closely related to other algebraic properties determined by the Algebraic-Analytic structure of C*-algebras. That is the case of 2-local and weak-2-local maps defined by 2-local actions of derivations and *-homomorphisms on C*-algebras.

We shall present the latest advances on 2-local and weak-2-local maps on C*-algebras. Let $\mathcal{S}$ be a subset of the space $L(X, Y)$ of all linear maps between Banach spaces X and Y. We recall that a (non-necessarily linear nor continuous) mapping $\Delta : X \to Y$ is a *2-local $\mathcal{S}$ map* (respectively, a *weak-2-local $\mathcal{S}$ map*) if for each $x, y \in X$ (respectively, if for each $x, y \in X$ and $\phi \in Y^*$), there exists $T_{x,y} \in \mathcal{S}$, depending on x and y (respectively, $T_{x,y,\phi} \in \mathcal{S}$, depending on x, y and ϕ), satisfying

$$\Delta(x) = T_{x,y}(x), \text{ and } \phi\Delta(y) = T_{x,y}(y)$$

(respectively,

$$\phi\Delta(x) = \phi T_{x,y,\phi}(x), \text{ and } \phi\Delta(y) = \phi T_{x,y,\phi}(y)).$$

We shall mainly focus on the case in which X and Y are C*-algebras and $\mathcal{S}$ is the set of all derivations on A or the set of all *-homomorphisms from A to B, or simply the set of all symmetric linear maps between A and B. We shall derive generalizations of well known, and significant, results due to Kowalski and Słodkowski [2], Šemrl [3], and Ayupov and Kudaybergenov [1].

References

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