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Banach spaces admitting many complemented subspaces

Manuel González

A Banach space X is called *subprojective* if every (closed) infinite-dimensional subspace of X contains an infinite-dimensional subspace complemented in X, and X is called *superprojective* if every infinite-codimensional subspace of X is contained in an infinite-codimensional subspace complemented in X. These two classes of spaces were introduced by Whitley [6] to find conditions for the conjugate of an operator to be strictly singular or strictly cosingular, and they were applied in [2] to obtain solutions to the perturbation classes problem for semi-Fredholm operators.

Banach spaces in which every subspace is complemented are isomorphic to Hilbert spaces [4], but a significant number of spaces are subprojective or superprojective. For example, $L_p(0,1)$ is subprojective if and only if $2 \leq p < \infty$, and it is superprojective if and only if $1 < p \leq 2$.

In this talk we describe some recent results on subprojective and superprojective spaces obtained in [5], [3] and [1].

References

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Departamento de Matemáticas, Universidad de Cantabria, Avenida de los Castros s/n, E-39071 Santander, Spain.
manuel.gonzalez@unican.es