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# On the non-triviality of certain spaces of analytic functions. Ultrahyperfunctions and hyperfunctions of fast growth.

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The test function spaces for Fourier ultrahyperfunctions and Fourier hyperfunctions (in one dimension) consist of functions $\varphi$ which are analytic on the horizontal strip $|\operatorname{Im} z| < k$ and satisfy

$$\sup_{|\operatorname{Im} z|<k} |\varphi(z)|e^{k|z|} < \infty,$$

for each $k > 0$ and some $k > 0$, respectively. In this talk we are interested in the following generalization of these spaces: let $M$ be a non-decreasing positive function defined on the positive half-axis satisfying certain natural conditions – e.g. the associated function of a weight sequence $M_p$ – and consider the spaces consisting of functions $\varphi$ which are analytic on the strip $|\operatorname{Im} z| < k$ and satisfy

$$\sup_{|\operatorname{Im} z|<k} |\varphi(z)|e^{M(k|z|)} < \infty,$$

for each $k > 0$ and some $k > 0$, respectively. Our results are twofold. Firstly, we present an analytic representation theory for the duals of these spaces and express them as cohomological quotients of spaces of analytic functions. Secondly, by using the aforementioned results, we characterize the non-triviality of these test function spaces in terms of the growth of the weight function $M$. In particular, we show that the Gelfand-Shilov spaces of Beurling type $\mathcal{S}^{(p!)}_{(M_p)}$ and Roumieu type $\mathcal{S}^{\{p!\}}_{\{M_p\}}$ are non-trivial if and only if

$$\sup_{p\geq 2} \frac{(\log p)^p}{h^p M_p} < \infty,$$

for all $h > 0$ and some $h > 0$, respectively.

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