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Superstable expansions of $(\mathbf{Z},+,0)$

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In [1] and in [2], it is proved that the structure $(\mathbf{Z},+,0,\Pi_q)$ is superstable, where Π_q is interpreted as the set of powers of a fixed natural number $q \geq 2$. In this work, we generalize this result to a wider class of sequences of natural numbers. Specifically, we prove that under one of the following two conditions on a sequence $R = (r_n)_{n\in\mathbf{N}}$ of natural numbers, the structure $(\mathbf{Z},+,0,R)$ is superstable:

1. $\lim_{n\to\infty} r_{n+1}/r_n = \infty$ and R is ultimately periodic modulo m, for any natural number $m > 1$;
2. R follows a linear reccurence relation and there exists a positive real number θ such that $\lim_{n\to\infty} r_n/\theta^n \in \mathbf{R}\backslash\{0\}$.

Our strategy is the following. We first give a universal axiomatization of the (first order) theory of $(\mathbf{Z},+,0,R)$ in a reasonable language so as to express the behaviour of equations satisfied by elements of R. We then prove that this theory is model-complete and, since our axiomatization is universal, deduce quantifier elimination. A counting of types argument then shows superstability.

References

[1] B. Poizat, Supergénérix, *Jounal of Algebra* **404** (2014), 240–270.

[2] D. Palacin and R. Sklinos, On superstable expansions of free abelian groups, *preprint* (2014),arXiv:1405.0568 .

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