

Some sums of powers of Catalan triangle numbers

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In this contribution we consider combinatorial numbers $C_{m,k}$, where

$$C_{m,k} := \frac{m-2k}{m} \binom{m}{k}, \quad m \geq 1, \quad k \geq 0,$$

which unifies the entries of the Catalan triangles $B_{n,k}$ and $A_{n,k}$ (see [2, 4]) for appropriate values of parameters m and k , i.e., $B_{n,k} = C_{2n,n-k}$ and $A_{n,k} = C_{2n+1,n+1-k}$. In fact, some of these numbers are the well-known Catalan numbers C_n that is

$$C_{2n,n-1} = C_{2n+1,n} = C_n = \frac{1}{n+1} \binom{2n}{n}, \quad n \geq 1.$$

We present new identities for recurrence relations, linear sums and alternating sum of $C_{m,k}$. After that, we check sums (and alternating sums) of squares and cubes of $C_{m,k}$ and, consequently, for $B_{n,k}$ and $A_{n,k}$. In particular, one of these equalities solves an open problem posed in [1]. We also present some linear identities involving harmonic numbers H_n and Catalan triangle numbers $C_{m,k}$. Finally, in the last section new open problems and identities involving C_n are conjectured. These results are collected in the preprint [3].

References

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