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Area estimates for constant mean curvature surfaces in $\mathbb{E}(\kappa, \tau)$-spaces

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Simply-connected Riemannian homogeneous 3-manifolds with isometry group of dimension 4 or 6, different from the hyperbolic space $\mathbb{H}^3$, are classified in a 2-parametric family $\mathbb{E}(\kappa, \tau)$, with $\kappa, \tau \in \mathbb{R}$. Surfaces with constant mean curvature $H \in \mathbb{R}$ (H-surfaces in the sequel) in these ambient spaces have been extensively studied in the literature, being some of the most celebrated results in this field are the existence of a holomorphic quadratic differential, obtained by Abresch and Rosenberg, and the solution of the Bernstein problem for H-surfaces whose curvature is critical (i.e., such that $4H^2 + \kappa = 0$) by Fernández and Mira (cf. [1]). The latter concerns the classification of surfaces which can be regarded as entire graphs in the direction of a unit Killing vector field in $\mathbb{E}(\kappa, \tau)$. The solution is based on a explicit relation between entire minimal graphs in Heisenberg space $\mathrm{Nil}_3(\tau) = \mathbb{E}(0, \tau)$ and holomorphic quadratic differentials in the complex plane or the unit disc, namely their Abresch-Rosenberg differential.

Though this is widely considered a satisfactory solution to the Bernstein problem, it does not give any insight into the geometry of such entire graphs. In the present work (cf. [2]) we deal with the study of the asymptotic growth of the area of H-graphs intersected with extrinsic or intrinsic metric balls when their radii tend to infinity. We will obtain some sharp upper bounds which can also be applied to understand the area growth of many other distinguished families of H-surfaces in $\mathbb{E}(\kappa, \tau)$-spaces, not necessarily graphs. Finally we will show that the estimates can be improved if additional assumptions on the growth of the height of the surface with respect to a given minimal section are assumed. Some open problems will be also discussed.

References

[1] I. Fernández and P. Mira, Holomorphic quadratic differentals and the Bernstein problem in Heisenberg space, *Trans. Amer. Math. Soc.* **361** (2011), 5737–5752.

[2] J. M. Manzano and B. Nelli, Height and area estimates for constant mean curvature graphs in $\mathbb{E}(\kappa, \tau)$-spaces. Preprint available in arXiv:1504.05239.

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