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Topological groups and spaces $C(X)$ with ordered bases

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An index set $\Sigma \subseteq \mathbb{N}^{\mathbb{N}}$ is boundedly complete if each bounded subset of Σ has an upper bound at Σ. If Σ is unbounded and directed (and if additionally Σ is boundedly complete) a base $\{U_\alpha : \alpha \in \Sigma\}$ of neighborhoods of the identity of a topological group G with $U_\beta \subseteq U_\alpha$, whenever $\alpha \leq \beta$ with $\alpha, \beta \in \Sigma$, is called in [2] a Σ-base (a long Σ-base). The case $\Sigma = \mathbb{N}^{\mathbb{N}}$ has been noticed for topological vector spaces under the name of $\mathfrak{G}$-base at [1]. If X is a separable, metrizable and not Polish space, the space $C_c(X)$ has a Σ-base but does not admit any $\mathfrak{G}$-base ([2]). Under an appropriate ZFC model the space $C_c(\omega_1)$ has a long Σ-base which is not a $\mathfrak{G}$-base ([2]).

In [2] we proved that (i) if G is a topological group with a long Σ-base then every compact subset of G is metrizable and (ii) that a Fréchet-Urysohn topological group is metrizable if and only if it has a long Σ-base. This result improve the recent result in [3] stating that a Fréchet-Urysohn topological group with $\mathfrak{G}$-base is metrizable.

By (i) if $C_c(X)$ has a long Σ-base then every compact subset of $C_c(X)$ is metrizable (i.e., $C_c(X)$ is strictly angelic). Then X is a C-Suslin space, and we get that $C_p(X)$ is angelic by Orihuela's theorem at [4], whence $C_c(X)$ is also angelic. Also we show in [2] that a $C_p(X)$ space has a long Σ-base if and only if X is countable.

Problem We do not know whether there exists a topological group with a long Σ-base that admits no $\mathfrak{G}$-base.

Problem Let X be a separable metric space admitting a compact ordered covering of X indexed by an unbounded and boundedly complete proper subset of $\mathbb{N}^{\mathbb{N}}$ that swallows the compact subsets of X. Is then X a Polish space?

References

[1] Cascales, B., Kąkol, J. and Saxon, S. A., *Metrizability vs. Fréchet-Urysohn property*, Proc. Amer. Math. Soc. **131** (2003), 3623–3631.

[2] Ferrando, J. C., Kąkol, J. and López-Pellicer, M. *Spaces $C(X)$ with ordered bases.* Submitted.

[3] Grabriyelyan, S., Kąkol, J. and Leiderman, A., *On topological groups with a small base and metrizability*, Fundamenta Math. **229** (2015), 129-158.

[4] Orihuela, J., *Pointwise compactness in spaces of continuous functions*, J. London Math. Soc. **36** (1987), 143-152.

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