II Joint Conference of the Belgian, Royal Spanish and Luxembourg Mathematical Societies
Logroño, June 6–8, 2016

A boundedness Nikodym property in algebras of Jordan measurable sets

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Let $ba(\mathcal{A})$ be the Banach space of the real (or complex) finitely additive measures of bounded variation defined on an algebra $\mathcal{A}$ of subsets of Ω endowed with the norm variation. A subset $\mathcal{B}$ of $\mathcal{A}$ has *property* N if a $\mathcal{B}$-pointwise bounded subset M of $ba(\mathcal{A})$ is bounded in $ba(\mathcal{A})$. $\mathcal{B}$ has *property* sN if for each increasing countable covering $(\mathcal{B}_m)_m$ of $\mathcal{B}$ there exists $\mathcal{B}_n$ which has property N and $\mathcal{B}$ has *property* wN if given the increasing countable coverings $(\mathcal{B}_{m_1})_{m_1}$ of $\mathcal{B}$ and $(\mathcal{B}_{m_1,m_2,\ldots,m_pm_{p+1}})_{m_{p+1}}$ of $\mathcal{B}_{m_1m_2\ldots m_p}$, for each $p, m_i \in \mathbb{N}$, $1 \leq i \leq p+1$, there exists a strand $(\mathcal{B}_{n_1n_2\ldots n_r}, r \in \mathbb{N})$ consisting of sets which have property N. The algebra of finite and co-finite subsets of $\mathbb{N}$ fails to have property N and Schachermayer proved that the algebra $\mathcal{J}(I)$ of Jordan measurable subsets of $I := [0,1]$ has property N and $\mathcal{J}(I)$ is not a σ-algebra. Valdivia proved in 2013 that the algebra $\mathcal{J}(K)$ of Jordan measurable subsets of a compact k-dimensional interval $K := \Pi\{[a_i, b_i] : 1 \leq i \leq k\}$ in $\mathbb{R}^k$ has property sN. We have proved that the algebra $\mathcal{J}(K)$ of Jordan measurable subsets of a compact subset K of a metric space has property wN. This result extends Schachermayer and Valdivia theorems and enables to give some applications to bounded vector measures.

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