

## Finite metric planes and related objects

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In the spirit of Hilbert’s axioms (cf. [4]), metric planes further generalise planar geometry through axioms solely relating to incidence, perpendicularity and reflections. The axioms allow for geometries beyond simply the Euclidean plane, including hyperbolic and elliptic geometries. However, Bachmann (cf. [1]) showed that in the finite case a metric plane is a Desarguesian affine plane of odd order, and conversely. Sherk (cf. [5]) generalised this result to characterise the finite affine planes of odd order by removing the ‘three reflections axioms’ from a metric plane. We consider these and the related partial Sherk planes (cf. [2]) with finitely many points. We will also show that partial Sherk planes characterise the Bruck nets (cf. [3]) of even degree.

### References

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