II Joint Conference of the Belgian, Royal Spanish and Luxembourg Mathematical Societies
Logroño, June 6–8, 2016

# Some variants of continuous Newton's method

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In this work we are concerned with some numerical properties of the differential equation

$$z(0) = z_0, \quad z'(t) = -\frac{p(z(t))}{p'(z(t))}, \tag{1}$$

where $z : \mathbb{R} \to \mathbb{C}$ and $p(z)$ is a given complex function. This problem is called continuous Newton's method and can be related with the classical Newton's method for solvingg nonlinear equations [1]. It is well-known ([2], [3]) that their solutions flow to the zeros of the $p(z)$ when $t \to \infty$. We show here that roots of multiplicity $m$ of the equation $p(z) = 0$ are asymptotically stable fixed points of the problem (1) with associate jacobian matrix given by

$$\begin{pmatrix} -1/m & 0 \\ 0 & -1/m \end{pmatrix}.$$

In addition, we explore some properties of the modified continuous Newton's method

$$z(0) = z_0, \quad z'(t) = -k\frac{p(z(t))}{p'(z(t))}, \quad k \in \mathbb{N}. \tag{2}$$

Actually we show that if $k_1 < k_2$ the solutions of (2) for $k_2$ flow to the roots of $p(z) = 0$ in a faster way than the solutions of (2) for $k_1$.

## References

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[2] J. Jacobsen, O. Lewis and B. Tennis, Approximations of continuous Newton's method: An extension of Cayley's problem, *Electron. J. Diff. Equ.* **15** (2007), 163–173.

[3] J. W. Neuberger, Continuous Newton's method for polynomials, *Math. Intell.* **21** (1999), 18–23.

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