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From elliptic functions to modular forms

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This work is part of Elliptic Functions and Elliptic Curves, a thesis to obtain the Master degree at Univ. Nac. Tucumán, Argentina.

From the standpoint of classical complex analysis, this work describes relationships between elliptic functions, elliptic curves and modular forms.

For each lattice Ω of $\mathbb{C}$ by using the Weierstass functions $\wp$ and $\wp'$ one defines an isomorphism between the complex torus associated to Ω and an elliptic curve in $\mathbb{C}P^2$.

It is shown that the space of modular forms $\mathcal{M}_{2k}$ is a direct sum of $\mathbb{C}G_{2k}$ and the space of cusp forms of weight $2k$ that are multiple of the discriminant function Δ a cusp form of minimum weight 12.

From the Fourier Series of G_{2k} at infinity, a normalization of Eisenstein's functions is obtained. The coefficients of that development involve Bernoulli numbers and the function $\sigma_{2k-1}(n)$, which is the sum of the $(2k-1)$-powers of the positive divisors of n. This illustrates the fact that the Fourier coefficients of modular forms are important arithmetic functions.

References

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