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Umbilicity properties of spacelike codimension two submanifolds

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Let $\Phi : (\mathcal{S}, g) \to (\mathcal{M}, \bar{g})$ be an isometric immersion of an n-dimensional Riemannian manifold into an $(n+2)$-dimensional semi-Riemannian manifold. $\mathcal{S}$ is said to be *umbilical* with respect to a normal vector ξ if the Weingarten operator A_ξ associated to ξ is proportional to the identity. We say $\mathcal{S}$ is *pseudo-umbilical* (*ortho-umbilical*) if it is umbilical with respect to the mean curvature H (by its orthogonal $\star^\perp H$).

We find necessary and sufficient conditions for $\mathcal{S}$ to be umbilical. These conditions are given in terms of algebraic properties of the Weingarten operators, or equivalently in terms of the trace-free part of the second fundamental form, that we call *total shear tensor*. We also find that if $\mathcal{S}$ is umbilical then the umbilical direction must be unique and we determine it explicitly.

If the ambient manifold $\mathcal{M}$ is Lorentzian, then $\mathcal{S}$ can be pseudo-umbilical and ortho-umbilical at the same time. This is possible when H is *null*, i.e. $\bar{g}(H, H) = 0$. We characterize this situation and present an example for $n = 2$.

This work generalizes to higher dimension and signature the results presented in (cf. [1]).

References

[1] J. M. M. Senovilla, Umbilical-Type Surfaces in Spacetime. In *Recent Trends in Lorentzian Geometry*, Springer Proceedings in Mathematics and Statistics, M. Sánchez et al. (eds.), pp. 87–109, Springer Science+Business Media, New York, 2013.

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