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Plane curves whose curvature depends on distance from a point

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The fundamental existence and uniqueness theorem in the theory of plane curves states that a curve is uniquely determined up to rigid motion by its curvature κ given as a function of its arc-length. However, in most cases such curves are impossible to find explicitly in practice, due to the difficulty in solving the quadratures appearing in the integration process. David A. Singer considered a different sort of problem (cf. [3]): *Can a plane curve be determined if its curvature is given in terms of its position?*

Singer started to deal with the posed problem by studying the condition $\kappa(r) = r := \sqrt{x^2+y^2}$, but only the very pleasant special case of the classical Bernoulli lemniscate, $r^2 = 3\cos 2\theta$ in polar coordinates, was solved explicitly by him (cf. [3]). Probably, the most interesting solved problem in this setting corresponds to the Euler elastic curves, whose curvature is proportional to one of the coordinate functions, say $\kappa(x,y) = c\,y$. Motivated by the above question and by the classical elasticae, the first two named authors recently studied the plane curves whose curvature depends on distance to a line (cf. [1]). We now study the plane curves whose curvature depends on distance from a point (say the origin, and so $\kappa = \kappa(r)$) requiring the computation of three quadratures (cf. [2]). In this way, we find out several interesting new families of spiral curves whose intrinsic equations are expressed in terms of elementary functions. We are able to get arc-length parametrizations of them and they are depicted graphically. We also provide new characterizations of some well known curves, like the Bernoulli lemniscate, the Norwich spiral and its inverse, the anti-clothoid or the cardioid.

References

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