II Joint Conference of the Belgian, Royal Spanish and Luxembourg Mathematical Societies
Logroño, June 6–8, 2016

Weighted Hardy inequalities, real interpolation methods and vector measures

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When the real interpolation method $(\cdot,\cdot)_{\rho,q}$ with a parameter function is applied to the pairs (L^1, L^∞) and $(L^{1,\infty}, L^\infty)$ of spaces with respect to a positive scalar measure, the result is a Lorentz space Λ^q_φ. Namely, when the parameter function ρ belongs to the class $Q(0,1)$, introduced by Persson in [3], it holds that (cf. [3, Proposition 6.2])

$$(L^1, L^\infty)_{\rho,q} = (L^{1,\infty}, L^\infty)_{\rho,q} = \Lambda^q_{\frac{t}{\rho(t)}}. \qquad (1)$$

In [1] we have established interpolation formulae for different pairs of spaces associated to a vector measure, providing in particular the corresponding version of (1) for the case of vector measures (cf. also [2]).

In this paper we continue the research started in [2] and [1], obtaining results that complements those ones. Now we are interested in analyzing the relationship between some conditions on the pair (ρ, q) and the K-spaces obtained by applying $(\cdot,\cdot)_{\rho,q,K}$ to the couples (L^1, L^∞) and $(L^{1,\infty}, L^\infty)$ of function spaces associated to the semivariation of a vector measure, when ρ is merely a positive measurable function defined on $(0,\infty)$. Note that for such a kind of functions, the equivalence theorem may fail, unlike it happens when $\rho \in Q(0,1)$. Our approach is based on the relationship of the pair (ρ, q) with a weighted Hardy type inequality for non-increasing functions.

References

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